Name of Lesson: *Parabolas and the Vertex Form of the Equation*

Mathematical Topic: Determining the behavior of parabolic functions given changes in the coefficients and variables in the vertex form of the equation.

Course: Algebra 2

Time Allocation: One (1) 56 minute period

Pre-requisite Knowledge: The students will be able to identify a parabolic function by inspection of a graphed function and understand where the vertex is located on the function. The students understand how to use the TI-84 handheld, can start applications and maneuver within applications and screens without teacher prompts or instruction.

Objectives: The students will:
1. Be able to identify the vertex form of the vertex equation of a parabola
2. Be able to explain what happens to the graph of the parabola when changes are made to the coefficients and variables of the equation
3. Be able to sketch parabolic functions given the equation of the function in vertex form
4. Be able to determine what the values of the coefficients and variables are in the vertex form of the equation given a graph of the function

Materials:
1. TI-84 Plus Silver Edition with the Guess My Coefficients Application (GuesCoef)
2. Graph paper and writing implement
3. TI View Screen and overhead projector.
4. Homework Answer Sheet (Overhead)
5. Activity Sheet 1
6. Activity Sheet 2

Important Vocabulary:

1. *Vertex*– the highest or lowest point of the figure depending upoin whether it opens up or down. It is also the point of intersection between the curve and its line of symmetry.
2. *Parabola* – a curve formed by the intersection of a cone and a plane parallel to one of the sides of the cone. The curve defined by the general equation $y = ax^n$ where n is a positive integer or fraction.
3. *Vertex Form of the Equation* – the form of the equation where the coordinate pair of the vertex is shown. It is of the form $y = a(x-h)^2 + k$

Procedure:

1. Homework Review (12 – 15 minutes)
   1.1 Place students in groups of threes using the counting method (1 – 2 – 3). Have the ones (1) get a piece of paper and act as the group's recorder. The students are to compare answers and help each other understand those for which they do not have agreement.. An answer sheet will be projected on the front screen. Any item that the group does not understand will be recorded on the paper along with the names of the students in the group. Students will remain in their groups for the entire class period.
   1.2 After 5 minutes (or so) have the student recorders come to the board and list the problems they were unable to do. The teacher matches groups (2 maximum) to help each other understand the problems listed on the board. Any problem which appears three or more times will be addressed by the teacher as some re-teaching is required.
   1.3 After another 5 minutes address any questions from the class on the homework.

2. Parabolas (30 – 35 minutes)
   2.1 Using the TI View Screen and TI-84 project y=x^2 on the screen for the students to see. Ask them to identify the function and describe it. Make sure to include how it opens (up), where the vertex is (0, 0) and something about end behavior. (The ends are both moving in a positive direction as it relates to y for example. You may be able to talk about how the y values are increasing as the left side's x value is decreasing and how the y value of the right side is increasing as the x value is increasing. Indicate that both are approaching infinity).
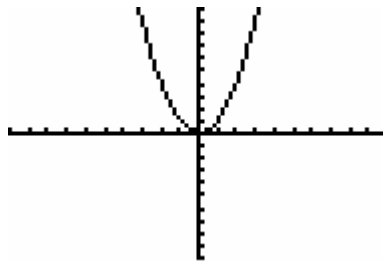

   2.2 Ask the students to get out their handhelds and enter y = x^2 into the y equation editor, then go to zoom and select option 6, Zoom Standard. Tell them that their graph should look like yours.
   2.3 Have the students use the trace function to find the vertex of the parabola. Ask then what they notice about the y value as they trace the parabola (it is never less than zero). Ask them to make a conjecture about the y value at the vertex. (it is zero)

2.4 Have the students go to table set and set the parameters as follows: (show this on the overhead)


```
TABLE SETUP
 TblStart=-1
 ΔTbl=.1
Indpnt: Auto Ask
Depend: Auto Ask
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2.5 Have the students go to the tables and then start scrolling through the values until they reach $x = 0$. Ask them what the y value is (zero). Ask them what the vertex of the equation is. The answer is the coordinate pair $(0, 0)$.

2.6 Ask the students to go the y – editor and change the equation to $y = 3x^2$ and graph the result. Ask them what effect the coefficient 3 had on the graph. (When compared to the first graph of $y = x^2$, the y values are increasing much more rapidly as the x values increase (on the right) and decrease (on the left).

2.7 Ask the students what they think will happen if the value is smaller than one (have them try $y = 0.5x^2$). They should indicate that the y values increase more slowly as x increases or decreases, depending upon the end you are observing.

2.8 Ask the students what they think will happen if you make the coefficient of x^2 negative. Have them try it. A graph of $y = -0.5x^2$ follows:
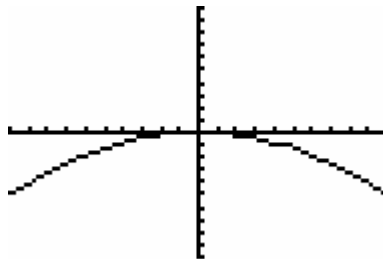
2.9 Ask the students to describe the change. (You may get answers like it flipped over or its upside down. We are looking for answers that indicate it is a reflection over the x-axis. Make sure they understand and if necessary verify that the vertex is still at $(0, 0)$)

2.10 Have the students go back to the y – editor and reenter y = x^2 and graph it. Ask them to observe the location of the vertex and the end behavior. Now have go back to the y –editor and enter the equations found on Activity Sheet 1 (Section 1) making sure to observe what changes to the graph of the parabola occur. Have them sketch each function on the graph paper and complete conjecture one on the activity sheet. (They should observe that changes in this value move the vertex of the parabola up and down along the y – axis, but do not change the end behavior.) The teacher should move around the room, observe student work and answer any questions that arise.

2.11 Have the students go back to the y – editor and enter the equations found in section 2 and observe the behavior of the function. Have them sketch the functions on the graph paper and complete observation 2. (They should observe that changes in this value move the vertex left and right). If students are not getting the correct results check the placement of parentheses and exponents in the equation.

2.12 Ask students to go to section 3 of the activity sheet and enter the equations into the y – editor. Sketch each on the graph paper and then complete observation 3. Ask the students what they observed with regard to the end behavior and the position of the vertex of the parabola as these values changed. In the context of their answers have them refer to the variables and coefficients using the following equation.

```
Plot1  Plot2  Plot3
\Y₁=A(X-H)²+K
\Y₂■
\Y₃=
\Y₄=
\Y₅=
\Y₆=
\Y₇=
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Answers should include the following:

A effects end behavior. The larger the value of A the faster y increases with respect to x. Additionally, if you change the sign of A, then you create a reflection of the graph. This is relative to whether it opens up or down.

H effects the movement of the vertex along the x axis; positive values of H move the vertex to the right equal to the value of H and negative values of H moves the vertex to the left equal to the value of H.

K effects the movement of the vertex along the y axis; positive values of K move the vertex up equal to the value of K and negative values of K move the vertex down equal to the value of K.

Note that observant students will recognize the H and K are the x and y coordinates of the parabola's vertex – hence the "name" of the equation.

2.13 Have the students complete observation 4 on the activity sheet.
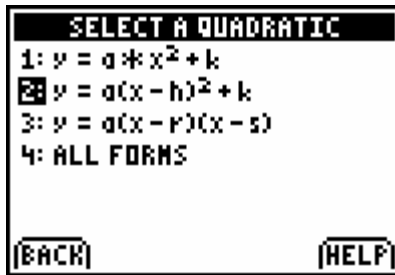
3. Putting It Together
    3.1 Hand out Activity Sheet 2, more graph paper and instruct the students to shut off their calculators and put them aside.
    3.2 Have the students do exercises 1 – 4. Each student must have their own graph of the equation. Ask the students to make the graphs as accurate as possible. They may work together.
    3.3 Review the student answers with them using the TI-84 and view screen. Have students make corrections to their work as needed. Now have them check the results with their calculators to ensure they can verify correct answers. Placement of the parentheses and the exponent are the most common mistakes when entering the equation. Students should use Zoom:6 to ensure they see enough of the function to make observations and verify work. Answer questions as needed.

4. Homework
    4.1 The homework is the balance of the exercises on Activity Sheet 2. The students will need their calculators to complete the homework and will be working with a new application, Guess My Coefficient. Ask the students to go to the APPS menu and select GuesCoef. Have them press enter until they come to the following screen:

Select options 2 (Quadratic) and then select option 2 on the next screen, which is the vertex form of the equation we have been working with.

```
SELECT A QUADRATIC
1: y = a * x² + k
2  y = a(x - h)² + k
3: y = a(x - r)(x - s)
4: ALL FORMS


[BACK]                [HELP]
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Press enter and the graph of a parabola will appear. Also appearing will be the number of chances you get to answer the problem and your current score. Press enter again to see just the graph and press enter a third time to get the "answer box". Enter the answers you think are correct for the values of the variables. Use the arrow key to move from one answer to the other. If you are stuck use the "hint" soft key. Press enter when you think you have the correct answer. The application will either trace the original graph (if you are correct) or draw the graph of the equation using your variables. It will then display a message indicating if you are right or wrong. Observe what happens and then use the OK hot key to proceed. When the answer box appears there will be a check mark next to any of the correct values. Change only those that are incorrect and then press enter. If you do not get the correct answer, the application will display it for you. (Make sure to press the OK hot key when prompted for the correct answer to be displayed) The teacher should be walking through this on the view screen as the students do it on their handhelds.

4.2 The balance of Activity Sheet 2 requires the students to go through two "complete" games of Guess My Coefficient and record the scores and more difficult problems on the activity sheet in the spaces provided. Following this they must complete the third section which is writing the equation of the parabola in vertex form given the coefficient "A" and the vertex point and then sketching it on graph paper. Calculators should be used to verify the results only.

Follow-up: Students who have mastered the material will be able to identify the vertex form of the vertex equation of a parabola, explain what happens to the graph of the parabola when changes are made to the coefficients and variables of the equation, sketch parabolic functions given the equation of the function in vertex form and determine what the values of the coefficients and variables are in the vertex form of the equation given a graph of the function. After successful completion of this lesson, students will be prepared to learn how cubics, quartics and quintics behave specifically, and have a better general knowledge of graph behavior for different types of functions. This also "sets the stage" for investigations involving conics later in the year.

Assessment: The homework activity sheet will be collected and corrected. The teacher will determine if the concepts presented have become part of the class's skill set before

moving on to more challenging problems. If the class has not mastered the material, additional practice using another handheld application (Transformation Graphing) and more worksheet problems with explanations will be employed.,

Resources: The activity (Guess My coefficient) is free from the TI website. (www.educatino.ti.com/us/propduct/apps/coefficients.html) All materials and the lesson activities are of my design.

ACTIVITY SHEET 1 – PARABOLAS

Name: _____ Date: _____ Block: _____

Section 1
Enter the following equations (one at a time) in the y editor of your handheld. Sketch the
graph for each on a piece of graph paper. Make sure to properly locate the vertex of the
curve and label the coordinate pair. The equation is in the form of $y = x^2 + k$.

a.) $y = x^2 - 4$ b.) $y = -x^2 + 1$

c.) $y = x^2 - 3$ d.) $y = -x^2 + 7$

Observation 1: What happens to the curve when the sign in front of "x" changes? What
happens to the curve when the values for "k" change?

Section 2
Enter the following equations (one at a time) in the y editor of your handheld. Sketch the
graph for each on a piece of graph paper. Make sure to properly locate the vertex of the
curve and label the coordinate pair. The equation is in the form of $y = (x – h)^2$.

e.) $y = (x-1)^2$ f.) $y = (x+3)^2$

g.) $y = -(x+4)^2$ h.) $y = (x-5)^2$

Observation 2: What happens to the vertex of the curve when the values for "h" change?

Section 3

Enter the following equations (one at a time) in the y editor of your handheld. Sketch the graph for each on a piece of graph paper. Make sure to properly locate the vertex of the curve and label the coordinate pair. The equation is in the form of $y = a(x-h)^2 + k$.

i.) $y = (x-4)^2 + 1$ j.) $y = -(x+2)^2 - 5$

k.) $y = 2(x+7)^2 + 5$ l.) $y = -3(x-6)^2 - 9$

Observation 3. Given the changes in the appearance of the parabola as the values for "a", "h", and "k" change, make some conjectures regarding how you might be able to graph a parabola given only this type of equation. (It is known as the VERTEX FORM OF THE EQUATION).

Observation 4. Write some specific rules on how to graph a parabola given the vertex form of the equation. Use the information in observation 3 along with what you have learned from class discussions.

ACTIVITY SHEET 2 – PARABOLAS

Name: _____ Date: _____ Block: _____

Section 1

Graph the equations which follow without the use of a calculator on the accompanying graph paper. Make the graph as accurate as possible and make sure to label the vertex. When you have completed all four, check your work with your calculator. If there are any errors, show the correct graph using a dotted line.

a. y = -4(x+3)^2 – 6   b. y = -(x+1)^2 + 1

c. y = 5(x+7)^2 +8)   d. y = (x-9)^2 -3

Section 2 Guess My Coefficient

1. From the APPS menu select "GuesCoef".
2. Press enter until you are shown a selection of games; select #2, Quadratic.
3. On the next screen select #2 again. (This is the vertex form of the equation we have been working with)
4. Press enter and the graph of a parabola will appear. Also appearing will be the number of chances you get to answer the problem and your current score.
5. Press enter again to see just the graph and press enter a third time to get the "answer box".
6. Enter the answers you think are correct for the values of the variables. Use the arrow key to move from one answer to the other. If you are stuck use the "hint" soft key. Press enter when you think you have the correct answer.
7. The application will either trace the original graph (if you are correct) or draw the graph of the equation using your variables (if you are incorrect). It will then display a message indicating if you are right or wrong.
8. If you are incorrect, observe what happens and then use the OK hot key to proceed. When the answer box appears there will be a check mark next to the correct values. Change only those that are incorrect and then press enter. If you still do not get the correct answer, the application will display it for you. (Make sure to press the OK hot key when prompted for the correct answer to be displayed)
9. Continue with the game until you get to the score page. Enter your score with your first and last initials and the number 1. (for example; PD1) This first game you played in class. You will now repeat the game twice more with the following additions to the "rules", starting with step 10..

10. After you have finished the first game and have recorded your score, use the hot key to "play again" and select the option 2, the vertex form of the equation.
11. As you are working through the second round, sketch the graph of any of the problems you did not answer correctly on the **second try** and write the accompanying equation below the graph. When finished with round two record your initials (i.e. PD2) and your score. Record your score on the graph paper as well.
12. Follow the instructions in #10 and play for a third time. This time record any problems along with their graphs which were not answered correctly the **first time.** Record your score and initials on the handheld (i.e. PD3) and on the graph paper.
13. Analyze your errors to see if there is a common theme to them, for example you might be off consistently on the y value of the vertex, or you might have the parabola opening up when it should open down. Use your calculator and Activity Sheet 1 as a guide to check the answers. Write a short explanation (using complete sentences) of what kinds of errors you were making and indicate what you have learned since performing the analysis that will enable you to answer problems like this correctly form this point forward. The answer to this question is to be put in the space below.