MAMT 590
Lesson Plan 3
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7/19/04

Name of Lesson: Pendulum Motion & Sine Waves

Mathematical Topics:
writing equations and determining coefficients for a general sine wave
curve fitting to data collected in an experiment using a motion detector

Course: Pre-Calculus or Advanced Algebra 2

Time Allocation: approximately 30 to 40 minutes

Pre-requisite Knowledge:
familiarity with graphing calculator lists, stat plots, and curve fitting
sine functions including amplitude, period, frequency, and phase shift

Objectives:
to practice collecting data using the CBL Motion Detector with a calculator
to learn how to interpret and fit a sine wave equation to experimental data

Materials & Equipment:
TI-83+ graphing calculator and the handout provided with instructions
CBL Motion Detector and CBL Unit with the necessary cables and manual
simple pendulum: a large ball suspended from at least one meter of string

Procedure:
Students should have already used the CBL equipment to do at least one activity in which they collected data on a calculator for a person or object moving in a straight line with constant velocity and then fit a linear equation to their data on a graph of distance versus time. In this activity they will perform similar tasks to collect data for the motion of a pendulum and then attempt to fit it to a sine wave equation.

Depending on availability of equipment and space as well as the size of the class, the students may collect their own data individually or the teacher may collect data for the entire class in a demonstration and then transfer it to the student calculators. Note: I have included a sample set of data on the next page to illustrate the results.

The students will analyze the data by trying to fit it with a sine wave equation of the form y = asin(bx + c) + d using the STAT PLOT feature of their calculators. After this they can try the automatic SinReg function to fit their data and compare the results, discussing the significance of each of the coefficients which appears in the equation and how it could be changed by modifying parts of the experiment.

Time	Distance	Time	Distance	Time	Distance
0.0	5.6829	3.4	2.0416	6.8	5.1146
0.2	5.9991	3.6	2.6129	7.0	3.9823
0.4	5.6170	3.8	3.6688	7.2	2.8561
0.6	4.6700	4.0	4.8403	7.4	2.1296
0.8	3.4889	4.2	5.7183	7.6	2.0564
1.0	2.4864	4.4	5.9961	7.8	2.6622
1.2	2.0126	4.6	5.5765	8.0	3.7353
1.4	2.2331	4.8	4.6062	8.2	4.9009
1.6	3.0708	5.0	3.4242	8.4	5.7518
1.8	4.2331	5.2	2.4433	8.6	5.9907
2.0	5.3140	5.4	2.0062	8.8	5.5342
2.2	5.9358	5.6	2.2656	9.0	4.5418
2.4	5.8815	5.8	3.1309	9.2	3.3601
2.6	5.1698	6.0	4.2998	9.4	2.4020
2.8	4.0496	6.2	5.3639	9.6	2.0020
3.0	2.9120	6.4	5.9516	9.8	2.3001
3.2	2.1544	6.6	5.8576	10.0	3.1919

Assessment:

Students will hand in at least one set of results for the experiment. They do not need to record the actual data, but they should show a sketch of the distance/time graph as well as a step-by-step explanation of how they fit a sine wave equation. They should also include the results of using the calculator's regression function SinReg and answer all the questions about the interpretation of their equations. For a follow-up homework assignment I would select an appropriate textbook exercise on writing and graphing general sine wave equations: y = asin(bx + c) + d.

Attachments:

worksheet with instructions, appropriate questions, and space for results

Resources/References:

TI-83+ graphing calculator guide and CBL Motion Detector/Unit manual

Pendulum Motion & Sine Waves

Name: ______________________________ **Date:** _______________

Instructions:

If you are collecting the data yourself or as part of a group, follow the instructions in the CBL manual to be sure that the CBL Unit, the CBL Motion Detector, and your graphing calculator are all connected properly. Be sure that the length of the pendulum is at least one meter and that it will swing freely toward and away from the motion detector, but not closer than half a meter. When you are ready, start the pendulum swinging and record a set of data until the motion detector has stopped.

If the teacher is conducting the demonstration, observe the procedure carefully and ask any questions if you do not understand why something is done a certain way. Once a good set of data has been collected, have it transferred to your calculator. The times will be in one list with the corresponding distances in another list.

Graph: Use the space below to sketch a distance/time scatter plot of the data from your calculator showing the time on the x-axis and the distance on the y-axis.

When you have finished your graph, you will try to fit a general sine wave function of the form $y = a\sin(bx + c) + d$ to your data using your calculator to verify results.

Analysis:

1. Write an equation $y = d$ for a horizontal line that appears to go through the center of the data you collected on your scatter plot: _______________

   What aspect of the experiment determines the size of the coefficient d?

   ____

   ____

2. Modify your equation $y = a\sin(x) + d$ to make a sine wave that appears to have the same amplitude a as the data you collected: _______________

   What aspect of the experiment determines the size of the coefficient a?

   ____

   ____

3. Include a frequency b in your equation $y = a\sin(bx) + d$ so that the sine wave is repeated the same number of times as your data: _______________

   What aspect of the experiment determines the size of the coefficient b?

   ____

   ____

4. Complete your equation $y = a\sin(bx + c) + d$ with a phase shift c so that the sine wave lines up as closely as possible with your data: _______________

   What aspect of the experiment determines the size of the coefficient c?

   ____

   ____

Use the SinReg function from the STAT/CALC menu on your calculator to find the sine wave regression equation of best fit for the data in your lists. Record both the coefficients a, b, c, and d along with the equation below:

Coefficients: ________________________ Equation: _________________

How does the equation you determined compare to the line of best fit?

____

____

Pendulum Motion & Sine Waves

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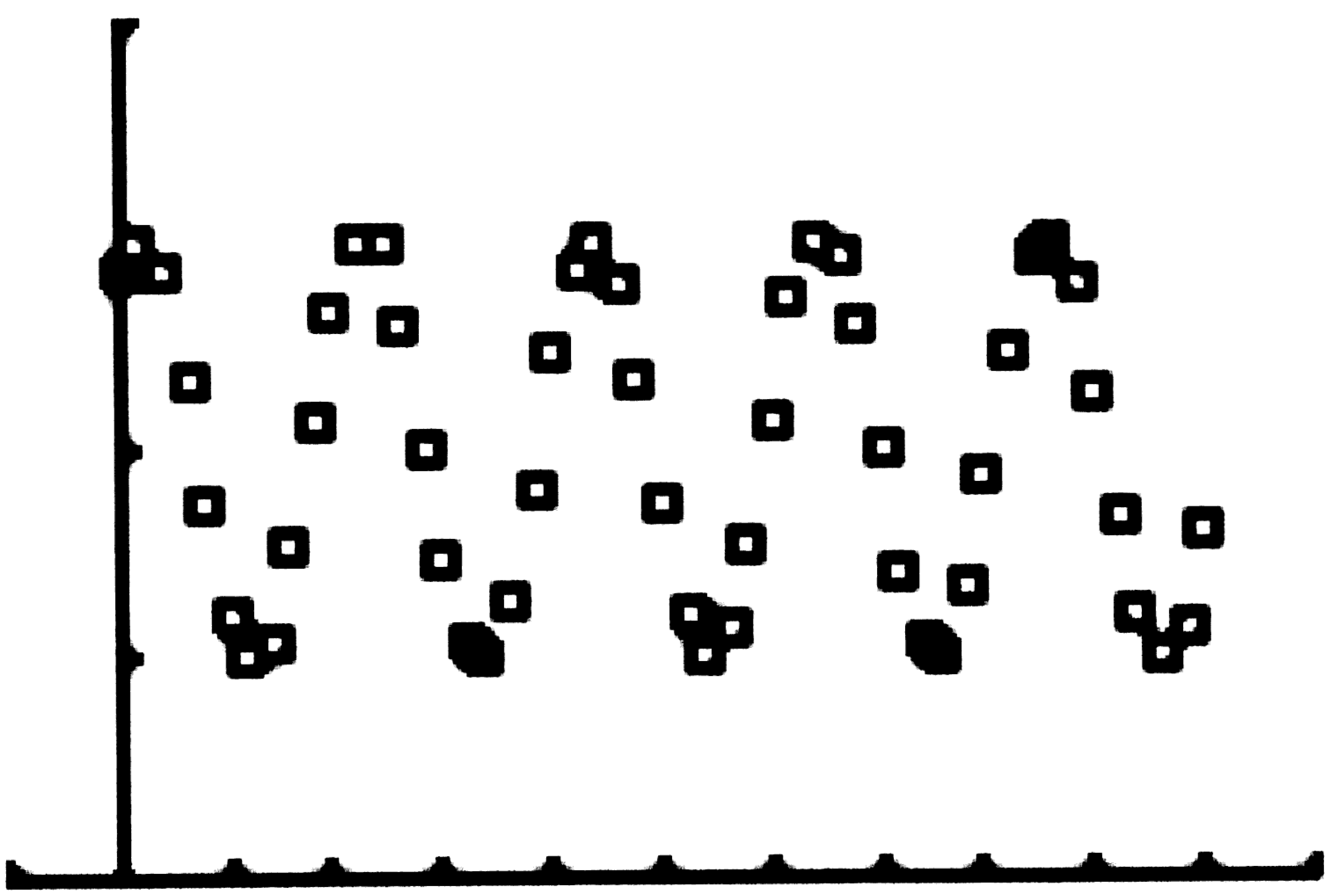
If the teacher is conducting the demonstration, observe the procedure carefully and ask any questions if you do not understand why something is done a certain way. Once a good set of data has been collected, have it transferred to your calculator. The times will be in one list with the corresponding distances in another list.

Graph: Use the space below to sketch a distance/time scatter plot of the data from your calculator showing the time on the x-axis and the distance on the y-axis.

When you have finished your graph, you will try to fit a general sine wave function of the form $y = a\sin(bx + c) + d$ to your data using your calculator to verify results.



Analysis:

1. Write an equation y = d for a horizontal line that appears to go through the center of the data you collected on your scatter plot: $y = 4$

   What aspect of the experiment determines the size of the coefficient d?
   The coefficient d is determined by the distance from the motion detector to the center point of the swing of the pendulum which is also its rest point.

2. Modify your equation $y = a\sin(x) + d$ to make a sine wave that appears to have the same amplitude a as the data you collected: $y = 2\sin(x) + 4$

   What aspect of the experiment determines the size of the coefficient a?
   The coefficient a is determined by how wide a swing the pendulum has as measured from the center of its swing toward and away from the detector.

3. Include a frequency b in your equation $y = a\sin(bx) + d$ so that the sine wave is repeated the same number of times as your data: $y =2\sin(3x) + 4$

   What aspect of the experiment determines the size of the coefficient b?
   The coefficient b is determined by the rate of swing of the pendulum which in turn is determined by the length of the string supporting it (optional).

4. Complete your equation $y = a\sin(bx + c) + d$ with a phase shift c so that the sine wave lines up as closely as possible with your data: $y =2\sin(3x + 1) + 4$

   What aspect of the experiment determines the size of the coefficient c?
   The coefficient c is determined by the position of the pendulum in its swing at the moment when the detector begins to record a particular set of data.

5. Use the SinReg function from the STAT/CALC menu on your calculator to find the sine wave regression equation of best fit for the data in your lists. Record both the coefficients a, b, c, and d along with the equation below:

   Coefficients: $a = 2$ $b = 3$ $c = 1$ $d = 4$ Equation: $y =2\sin(3x + 1) + 4$

   How does the equation you determined compare to the line of best fit?
   The SinReg function on the calculator produces exactly the same equation and coefficients for a sine wave that fits the data within appropriate limits.